Description of code to process cup and rim profile results

1. Scale data and down-sample.
2. Locate datum for each data set.
3. Match datum coordinates for all data sets to a reference set.
4. Rotate data sets about the datum to the reference set.
5. Rotate all sets to horizontal plane.
6. Calculate the distance between the reference set and all other data sets between two locations.
7. Determine the area between the reference set and all other data sets.
8. Determine the tangential direction at a location on the reference set.
9. Calculate the normal direction and where the resulting scaled vector intersects all other data sets.
10. Solve for the magnitude of the normal distance between the reference set and all other data sets.

NOTE: At the start of each step the updated coordinates $x' y'$ become the current $x$,$ y$ coordinates.

1. Scale data and down-sample.

The raw $X$, $Y$ data is scaled for each data set by a factor $S\_{f}$ = 1000:

$x=\frac{X}{S\_{f}}$ $ y=\frac{Y}{S\_{f}}$

The scaled $x$,$ y$ data is down-sampled to reduce the noise of the discrete response. Every 1 in $n$ data points is kept, where $n$ = 100 and $N$ is the total number of points in the data set before down-sampling. The total number of points in the down-sampled set $N/n$ is rounded to ensure that there is no discontinuity when sampling:

$$x'\_{i},y'\_{i}=x\_{in},y\_{in} i=1,…,N/n$$

2. Locate the datum for each data set.

Defining the datum for each data set works by the principle of finding where the $y$ values upstream and downstream of a specific $x$ location differ by a significant amount. The code iterates along the $x$ value of the data set from just upstream of the datum ($p$ = 10 data points), the mean $y$ value of the $r$ = 10 points upstream and downstream of this location are determined. If the upstream mean is less than a specific multiple of the downstream then the code terminates and the corresponding $x$,$ y$ value is given as the datum location. The value assigned to the specific multiple is by default 0.5, this may need to be adjusted depending on the initial gradient of the data sets provided. Mathematically the datum is given by initially satisfying the inequality for $j$:

$$\sum\_{i=1}^{r}y\_{p+i+j}<0.5\sum\_{i=1}^{r}y\_{p-i+j} j=1,…,N/n$$

3. Match datum coordinates for all data sets.

One data set is selected at the reference set $x^{ref}$,$ y^{ref}$. All other data sets are translated so that the datum locations determined in part 2 $x\_{0}$,$ y\_{0}$ are the same as the reference set:

$$x^{'}=x-x\_{0}+x\_{0}^{ref}$$

$$y^{'}=y-y\_{0}+y\_{0}^{ref}$$

4. Rotate data sets about the datum to the reference set.

The data sets must all be rotated about the common datum so that the angles between the edges protruding downstream of the datum are zero. The edge of the reference set is chosen as the common plane, all data sets are linear over the upstream region close to the datum thus ensuring the angles can be calculated using vectors. A location upstream is chosen for the reference set $x\_{A}$,$ y\_{A}$ and each of the other data sets $x\_{B}$,$ y\_{B}$, the angle between the two edges $θ$ can be calculated by the following:

$$a=\left(\begin{matrix}x\_{A}-x\_{0}\\y\_{A}-y\_{0}\end{matrix}\right) b=\left(\begin{matrix}x\_{B}-x\_{0}\\y\_{B}-y\_{0}\end{matrix}\right)$$

$$\cos(θ)=\frac{a∙b}{\left|a\right|\left|b\right|}$$

0

θ

**a**

**b**

A

B

Once the angles $θ$ have been determined the coordinate rotation of each of the data sets to the reference set can be calculated using the matrix equation as follows:

$$\left(\begin{matrix}x^{'}\\y'\end{matrix}\right)=\left(\begin{matrix}\cos(θ)&-\sin(θ)\\\sin(θ)&\cos(θ)\end{matrix}\right)\left(\begin{matrix}x\\y\end{matrix}\right)$$

5. Rotate all sets to horizontal plane.

The common plane chosen in part 4 needs to be rotated such that it is horizontal, this ensures that all data sets line up to the horizontal plane in the region just upstream of the datum. The angle between the reference plane and horizontal axis $θ'$ is determined by:

$$\tan(θ')=\frac{y\_{A}}{x\_{A}}$$

y

x

0

θ'

A

Once the angle $θ'$ has been determined the coordinate rotation of each of the data sets to the horizontal plane can be calculated using the matrix equation as follows:

$$\left(\begin{matrix}x^{'}\\y'\end{matrix}\right)=\left(\begin{matrix}\cos(θ')&-\sin(θ')\\\sin(θ')&\cos(θ')\end{matrix}\right)\left(\begin{matrix}x\\y\end{matrix}\right)$$

6. Determine the tangential direction at any location on the reference set.

At any specified location $x\_{C}$,$ y\_{C}$ the normal distance from the reference set to all other data sets is required $∆n$, the first step of this is to calculate the tangential direction at any location on the reference set. This process assumes that over a small number of discrete points near the specific location a vector between the limits of this region represents the tangential direction. This assumption holds so long as the region is small and the gradient of the curve does not change significantly, which is the case in this problem. Taking $s$ = 5 points upstream and downstream of $x\_{N}$ the tangential unit vector $\hat{t} $to the reference set can be determined:

$$\hat{t}=\frac{v\_{ds}-v\_{us}}{\left‖v\_{ds}-v\_{us}\right‖}=\left(\begin{matrix}x\_{\hat{t}}\\y\_{\hat{t}}\end{matrix}\right)$$

$$v\_{ds}=\left(\begin{matrix}x\_{C+s}-x\_{0}\\y\_{C+s}-y\_{0}\end{matrix}\right)$$

$$v\_{us}=\left(\begin{matrix}x\_{C-s}-x\_{0}\\y\_{C-s}-y\_{0}\end{matrix}\right)$$

0

$$y^{ref}=f\left(x^{ref}\right)$$

$$v\_{us}$$

$$v\_{ds}$$

$$\hat{t}$$

$$\hat{n}$$

C

7. Calculate the normal direction and where the resulting scaled vector intersects all other data sets.

Having determined the tangential unit vector at a specific location along the reference set the normal unit vector can be calculated:

$$\hat{n}=\left(\begin{matrix}-y\_{\hat{t}}\\x\_{\hat{t}}\end{matrix}\right)$$

Knowing the normal unit vector at any location of the reference set means that the location along the other data sets where this vector intersects can also be determined. Scaling the unit vector by a factor of $k$ produces a vector normal to the reference set $n$, the $x$ location where the $y$ values of this vector and the remaining data sets are zero gives the intercept. This is achieved numerically by determining the minimum value of the absolute error between the two over a range of $x$ with $M$ discrete locations.

$$n=k\hat{n}=\left(\begin{matrix}x\_{n}\\y\_{n}\end{matrix}\right)$$

$$\min\_{x\_{n,i}}\left|y\_{n,i}-\tilde{y}\_{i}\right| i=1,…,M$$

$$\tilde{y}=\tilde{f}(x,y,x\_{n})$$

8. Solve for the magnitude of the normal distance between the reference set and all other data sets. The range of data points in which $∆n$ is calculated is given by the datum and a pre-determined downstream y value (at which the profile no longer represents the rim region).

Once the value of $x\_{n}$ which gives the minimum error is determined from part 9 the intercept location $x\_{D}$,$ y\_{D}$ is known. Using vectors the magnitude of the vector $n$, $∆n$ can be calculated:

$$∆n=\left‖c-d\right‖$$

$$c=\left(\begin{matrix}x\_{C}-x\_{0}\\y\_{C}-y\_{0}\end{matrix}\right)$$

$$d=\left(\begin{matrix}x\_{D}-x\_{0}\\y\_{D}-y\_{0}\end{matrix}\right)$$

0

$$y^{ref}=f\left(x^{ref}\right)$$

**c**

$$y=f\left(x\right)$$

**d**

$$∆n$$

C

$$\hat{t}$$

$$\hat{n}$$

D

Using the values of $∆n$ calculated a numerical procedure identifies the maximum normal distance between the two data sets and also gives the location at which this occurs.